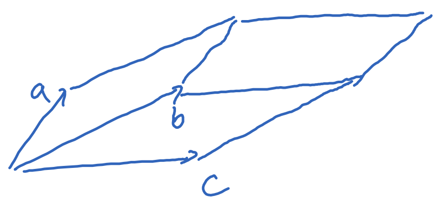
* Triple product: $a \cdot (b \times c)$

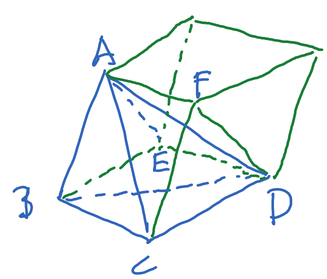
Another representation: $a \cdot (b \times c) = \det \begin{bmatrix} — a — \\ — b — \\ — c — \end{bmatrix} = \det \begin{bmatrix} | & | & | \\ a & b & c \\ | & | & | \end{bmatrix}$



Geometrically,

$|a \cdot (b \times c)|$ = volume of the parallelopiped formed by a, b, c.

Ex



$$\underbrace{vol(ABCD) = vol(ABDE) = vol(ACDF)}_{\text{sum} = \frac{1}{2} \text{ volume of parallelopiped}}$$

A(1, 0, 0)

B(2, 1, 0)

C(3, 1, 1)

D(0, 2, -1)

Thus,

$$vol(ABCD) = \frac{1}{6} vol(\text{parallelopiped})$$

$$= \frac{1}{6} \left| \vec{AB} \cdot (\vec{AC} \times \vec{AD}) \right|$$

$$= \frac{1}{6} \left| \langle 1, 1, 0 \rangle \cdot (\langle 2, 1, 1 \rangle \times \langle -1, 2, -1 \rangle) \right|$$

$$\begin{matrix} 2 & 1 & 1 & 2 & 1 \\ -1 & 2 & -1 & -1 & 2 \end{matrix}$$

$$= \frac{1}{6} \left| \langle 1, 1, 0 \rangle \cdot \langle -3, 1, 5 \rangle \right|$$

$$= \frac{1}{6} |-3 + 1 + 0| = \frac{1}{3}$$